

Non-Commutative Rings and their Applications
NCRA, VII
Lens, France
5-7th July 2021

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Modules with the Exchange Property

Abstract: A right R -module M is said to satisfy the (full) exchange property if for any two direct sum decompositions $L = M \oplus N = \bigoplus_{i \in I} N_i$, there exist submodules $K_i \subseteq N_i$ such that $L = M \oplus (\bigoplus_{i \in I} K_i)$. If this holds only for $|I| < \infty$, then M is said to satisfy the finite exchange property. A ring R for which R_R has the finite exchange property is called an exchange ring. The exchange property is of importance because it provides a way to build isomorphic refinements of different direct sum decompositions, which is precisely what is needed to prove the famous Krull-Schmidt-Remak-Azumaya Theorem.

It is an open question due to Crawley and Jónsson whether the finite exchange property always implies the full exchange property. This question was provided a positive answer for modules with indecomposable decompositions by Zimmermann-Huisgen & Zimmermann, for quasi-injective modules by L. Fuchs, for auto-invariant Modules by Guil Assensio and Srivastava, and for Square-free modules by P. Nielsen. In this talk we provide new classes of modules, where the finite exchange implies the full exchange. This is a joint work with Yasser Ibrahim of both Taibah and Cairo Universities.